THEOREMS OF PROBABILITY – Prof. Richard B. Goldstein

CHEBYCHEV'S THEOREM

At least $\left(1-\frac{1}{k^2}\right) \bullet 100$ *percent of any set of data falls within k standard deviations of the mean.*

$$\boxed{\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}}$$

LAW OF LARGE NUMBERS / BERNOULLI'S THEOREM

If the number of times a situation is repeated becomes larger and larger, the proportion of successes will tend to come closer and closer to the actual probability of success.

$$\boxed{\text{For any } \varepsilon > 0 \;\; \lim_{n\to\infty} \mathrm{P}\left\{\left|\frac{x}{n} - p\right| < \varepsilon\right\} = 1 \;\; \text{where } x = \#\text{of successes in } n \text{ trials}}$$

More generally,

$$\boxed{\lim_{n\to\infty} \mathrm{P}\left\{\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| < \varepsilon\right\} = 1}$$

where $\{X_k\}$ are identical, mutually independent random variables with $\mu = E(X_k)$.

CENTRAL LIMIT THEOREM / DE MOIVRE & LAPLACE

The sum of n random numbers becomes more and more like a normal distribution.

Let $\{X_k\}$ be a set of identical, mutually independent random variables
Let $\mu = E(X_k)$ and $\sigma^2 = Var(X_k)$ for all k. Then,

$$\boxed{\mathrm{P}\left\{\frac{S - n\mu}{\sigma\sqrt{n}} < t\right\} = \mathrm{P}\left\{\frac{\overline{x} - \mu}{\sigma / \sqrt{n}} < t\right\} \to \Phi(t)}$$

where $S = X_1 + X_2 + \ldots + X_n$, and $\Phi(t)$ = cumulative standard normal distribution.

Note: The theorem has been extended to cases where the distributions are not identical and also when they are not independent.