Math 523 - Prof. Richard B. Goldstein - Some Common Probability Examples

DICE

Two dice (36 in sample space)

sum	2	3	4	5	6	7	8	9	10	11	12
frequency	1	2	3	4	5	6	5	4	3	2	1

Three dice (216 in sample space)

sum	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
frequency	1	3	6	10	15*	21	25	27	27	25	21	15	10	6	3	1

- 1-1-5 has 3 arrangements, 1-2-4 has 6, 1-3-3 has 3, and 2-2-3 has 3

CARDS

Royal Flush	A K Q J 10 all the same suit	4
Straight Flush	Five cards in a sequence and of the same suit, but not a royal flush	36
Four of a kind	Four cards of the same denomination	624
Full house	Three of one denomination and a pair of a different denomination ${}_4C_3 * {}_4C_2 * 13 * 12 = 3744$	3744
Flush	Five cards of the same suit $({}_{13}C_5 - 10)*4 = 5108$ (not in a sequence)	5108
Straight	Five cards in a sequence but not the same suit	10200
Three of a kind	Three cards of the same denomination	54912
Two pairs	Two pairs each with different denomination	123552
One pair	A single pair	1098240
Nothing	None of the above	<u>1302540</u>
		2598960

LOTTERY

Suppose there are N numbers and n have to be chosen exactly.

Example: N = 44 n =6

There are ${}_{44}C_6$ = 7,059,052 possible choices

${}_6C_6 * {}_{38}C_0 = 1$	all 6	winning ticket
${}_6C_5 * {}_{38}C_1 = 228$	5 of 6	second prizes
${}_6C_4 * {}_{38}C_2 = 10{,}545$	4 of 6	third prizes
${}_6C_3 * {}_{38}C_3 = 168{,}720$	3 of 6	
${}_6C_2 * {}_{38}C_4 = 1{,}107{,}225$	2 of 6	
${}_6C_1 * {}_{38}C_5 = 3{,}011{,}652$	1 of 6	
${}_6C_0 * {}_{38}C_6 = 2{,}760{,}681$	0 of 6	

Power Ball

Extra number from 1 to M

${}_NC_n * M$ possible choices

Keno

There are 80 numbers and 20 are chosen. You can choose from 1 to 15 different numbers from 1 to 80 and the more you match the more $ you win.

Example: you choose 8 numbers. The payoffs on $ are as follows:

winning spots	$1 pays
4	$4.00
5	8.00
6	40.00
7	400.00
8	10,000.00

There are ${}_{80}C_{20} = 3.535316142 \text{ X } 10^{18}$ possible ways of choosing all 20 numbers
There are ${}_{80}C_8$ = 28,987,537,150 ways for you to choose 8 numbers

4 spots:	${}_{20}C_4 * {}_{60}C_4 = 2{,}362{,}591{,}585$	possibilities; prob. = 0.0815037
5 spots:	${}_{20}C_5 * {}_{60}C_3 = 530{,}546{,}880$	possibilities; prob. = 0.0183026
6 spots:	${}_{20}C_6 * {}_{60}C_2 = 68{,}605{,}200$	possibilities; prob. = 0.0023667
7 spots:	${}_{20}C_7 * {}_{60}C_1 = 4{,}651{,}200$	possibilities; prob. = 0.0001605
8 spots:	${}_{20}C_8 * {}_{60}C_0 = 125{,}970$	possibilities; prob. = 0.0000043

The expected payoff is $0.6747 or a profit of -$0.3253