Prof. Richard B. Goldstein – Discrete and Continuous Probability Distributions

Distribution	f(x)	mean	variance	practical uses
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$ x = 0, 1, 2, …, n	np	np(1-p)	x successes out of n trials
Geometric	$p(1-p)^{x-1}$ x = 1, 2, 3, …	$\frac{1}{p}$	$\frac{(1-p)}{p^2}$	first success occurs on trial #x
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2, \ldots$	λ	λ	x successes given λ were expected

Hypergeometric	$\frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N_1+N_2}{n}}$ $x = 0, 1, \ldots, \min(n, N_1)$	$\frac{nN_1}{N_1+N_2}$	$n\left(\frac{N_1}{N_1+N_2}\right)\left(\frac{N_2}{N_1+N_2}\right)\left(\frac{N_1+N_2-n}{N_1+N_2-1}\right)$	N_1 = # successes
				N_2 = # failures
				n = # chosen
				x = # successes from chosen

Negative Binomial	$\binom{x+s-1}{s-1} p^s (1-p)^x$ x = 0, 1, 2, … $s \geq 1$	$\frac{s(1-p)}{p}$	$\frac{s(1-p)}{p^2}$	success #s occurs after x failures
Multinomial	$\frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$ where $\sum_{i=1}^{k} x_i = n$ and $\sum_{i=1}^{k} p_i = 1$	$E[x_i]=np_i$		Each trial has k possible outcomes $E_1, E_2, \ldots, E_k$

Note: The Geometric distribution is a special case of the Negative Binomial where s = 1.
The Binomial distribution is a special case of the Multinomial where k = 2.

Distribution	f(x)	mean	variance	practical uses
Beta	$x^{z-1}(1-x)^{w-1}/\beta(z,w)$ on $[0,1]$	$\frac{z}{z+w}$	$\frac{zw}{(z+w)^2(z+w+1)}$	proportions
Chi-Square	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$ on $[0,\infty)$	n	2n	goodness-of-fit tests
Exponential	$\lambda e^{-\lambda x}$ on $[0,\infty)$	$1/\lambda$	$1/\lambda^2$	queues, memory-less
F	$\frac{n_1^{n_1/2}n_2^{n_2/2}}{\beta(n_1,n_2)}x^{\frac{n_1-2}{2}}(n_2+n_1x)^{-\frac{n_1+n_2}{2}}$ on $[0,\infty)$	$\frac{n_2}{n_2-2}$	complicated	hypothesis tests comparing two variances
Gamma	$\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{(\alpha-1)}e^{-\lambda x}$ on $[0,\infty)$	α/λ	α/λ^2	generalized exponential
Log-Normal	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$ on $[0,\infty)$	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2\mu}(e^{2\sigma^2}-e^{\sigma^2})$	price behavior of financial instruments
Normal	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ on $(-\infty,\infty)$	μ	σ^2	standard bell-shaped distribution used on errors, test scores
Student-T	$\frac{1}{\sqrt{n}\beta(\frac{1}{2},\frac{n}{2})}(1+\frac{x^2}{n})^{-\frac{n+1}{2}}$ on $(-\infty,\infty)$	0	$\frac{n}{n-2}$ if $n>2$	hypothesis tests comparing means
Uniform	$\frac{1}{b-a}$ on $[a,b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	random numbers on [0, 1]
Weibull	$\alpha\beta^\alpha x^{\alpha-1}e^{-(\beta x)^\alpha}$ on $[0,\infty)$	$\frac{1}{\beta}\Gamma(\frac{1}{\alpha}+1)$	$\frac{1}{\beta^2}(\Gamma(\frac{2}{\alpha}+1)-\Gamma^2(\frac{1}{\alpha}+1))$	time to failure of components or systems in reliability problems

Johnson SB: $\frac{\alpha_2}{x(1-x)\sqrt{2\pi}}e^{-\frac{\left[\alpha_1+\alpha_2\ln\left(\frac{x}{1-x}\right)\right]^2}{2}}$ on $[0,1]$ Johnson SU: $\frac{\alpha_2}{\sqrt{2\pi}\sqrt{x^2+1}}e^{-\frac{\left[\alpha_1+\alpha_2\ln\left(x+\sqrt{x^2+1}\right)\right]^2}{2}}$ on $(-\infty,\infty)$

and the log-normal are a family of transforms of the Normal used for fitting.