PROBABILITY – Prof. Richard B. Goldstein

Definitions

Classical / Theoretical – sample space, outcomes, events $P(A) = \dfrac{n(A)}{n(S)}$

Experimental – frequency of event $P(A) = \dfrac{\#\text{ of occurrences of A}}{\#\text{ of trials of experiment}}$

Subjective – estimated by the individual

Set Notation

$P(A)$ = probability of an event A
$P(A') = 1 - P(A)$: complement of A (not A)
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ - show Venn Diagrams
$P(A' \cap B)$, $P(A' \cap B \cap C')$
$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{n(A \cap B)}{n(B)}$
Independent / Dependent
All = vs. ≠ : $P(A \mid B) = P(A)$, $P(B \mid A) = P(B)$, $P(A \cap B) = P(A)\,P(B)$

Counting Rules

Multiplication Rule: $n_1 \times n_2 \times \ldots \times n_k$
Factorial: $n! = 1 \times 2 \times 3 \times \ldots \times n$
Permutations: ${}_nP_r = \dfrac{n!}{(n-r)!}$
Combinations: ${}_nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{{}_nP_r}{r!}$ (Pascal's Triangle)
Multinomial: $\dbinom{n}{n_1, n_2, \ldots, n_r} = \dfrac{n!}{n_1! n_2! \cdots n_r!}$ where $n_1 + n_2 + \ldots + n_r = n$
Tree Diagrams

Multiplication Rules: $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2)$
$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$ if independent

Bayes' Rule

$$P(B_r \mid A) = \frac{P(B_r \cap A)}{P(A)} = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A \mid B_r)}{\sum_{i=1}^{k} P(B_i)P(A \mid B_i)} \quad \text{for } r = 1, 2, \ldots, k$$