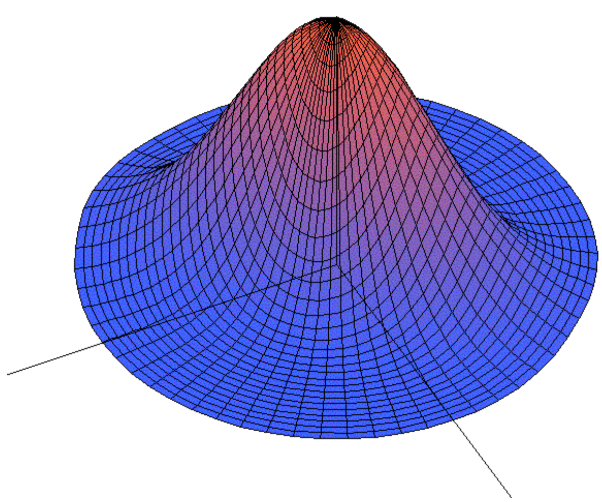
Multivariate Normal Distribution

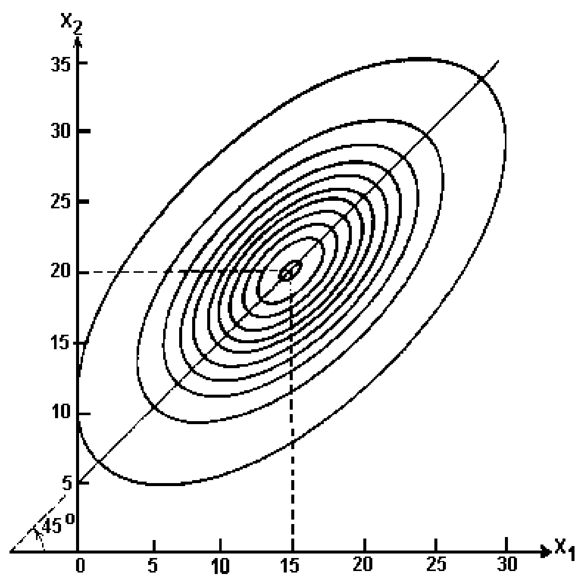
$$f(x,y) = \frac{1}{2ps_1 s_2\sqrt{1-r^2}}\exp\left[-\frac{1}{2(1-r^2)}\left\{\frac{(x-m_1)^2}{s_1^2}+\frac{(y-m_2)^2}{s_2^2}-2r\frac{(x-m_1)(y-m_2)}{s_1 s_2}\right\}\right]$$

$\frac{(x-m_1)^2}{s_1^2}+\frac{(y-m_2)^2}{s_2^2}-2r\frac{(x-m_1)(y-m_2)}{s_1 s_2} = C$ represents an ellipse with center at the point (μ_1, μ_2) which is called the *centroid* of the bivariate population, and major or minor axis along the line passing through this point and making the following angle with the positive x-axis:

$$\theta = \begin{cases} \frac{1}{2}\,\mathrm{Arc}\tan\frac{2\rho\sigma_1\sigma_2}{\sigma_1^2-\sigma_2^2} & \text{when } \sigma_1 \neq \sigma_2 \\ 45^{o} & \text{when } \sigma_1 = \sigma_2 \end{cases}$$

. This line contains the major axis if $\rho > 0$ and the minor axis if $\rho < 0$.





Several members of the family of ellipses:
$(x - 15)^2 + (y – 20)^2 – 2(0.6)(x – 15)(y – 20) = C$

In general, for n dimensions $f(x_1, x_2, \ldots, x_n) = (2\pi)^{-n/2}|\Sigma|^{-1/2}\exp(-\chi^2/2)$ where

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & & \ddots & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \rho_{n2}\sigma_n\sigma_2 & \cdots & \sigma_n^2 \end{bmatrix},\ \chi^2 = X'\Sigma^{-1}X, \text{ and } X = \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \\ \vdots \\ x_n-\mu_n \end{bmatrix}$$