Hypothesis Testing

Null Hypothesis H_0 statement to be tested; examples $H_0 : \mu = \mu_0$, $H_0 : p_1 - p_2 = 0$

Alternative Hypothesis H_1 the statement that will be adopted if there is strong, significant evidence from the data to reject the null hypothesis; examples $H_1 : \mu > \mu_0$, $H_1 : p_1 - p_2 \neq 0$

Our Decision

Truth of H_0	**We accept H_0 as true**	**We reject H_0 as false**
H_0 is true	Correct decision with prob. $1 - \alpha$	Type I error with prob. α
H_0 is false	Type II error with prob. β	Correct decision with prob. $1 - \beta$

α is the level of significance; typically set in advance as 5% or 1%

$1 - \beta$ is the power of a test and represents the probability of rejecting H_0, when it is, in fact, false

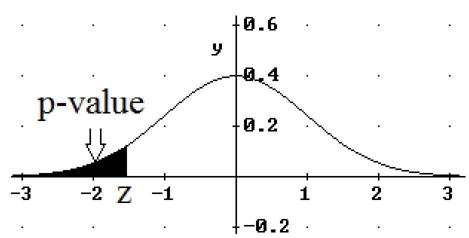
p – value: given that H_0 is true this is the probability that the test statistic will take on values as extreme or more extreme than the observed value based upon the sample data

<u>Steps</u>

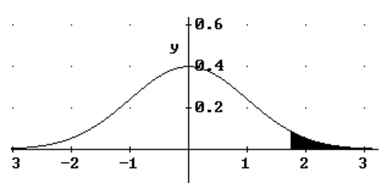
[1] Determine the null hypothesis, H_0, the alternative hypothesis, H_1, and the level of significance, α
[2] Select the test statistic – for example z, t, χ^2, F, etc.
[3] Calculate the p-value based upon the sample and the test statistic used
[4] Conclusion: If p-value $\leq \alpha$, we reject H_0 and if p-value $> \alpha$, we do not reject H_0
[5] Interpretation of the test results

Example: $H_0 : \mu = \mu_0$ Test statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

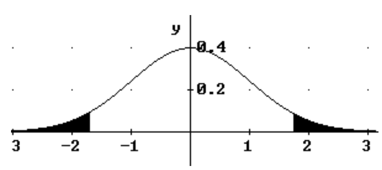
$H_1 : \mu < \mu_0$ (left-tailed)



Alternative: $H_1 : \mu > \mu_0$ (right-tailed)



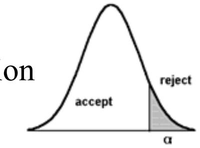
Alternative: $H_1 : \mu \neq \mu_0$ (two-tailed)



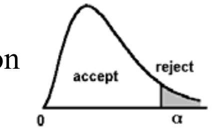
p = the combined shaded areas

Prof. Richard B. Goldstein – Statistics

Normal or Student-t Distribution



Chi-Square or F Distribution



Parameter	Test	Hypotheses	Test Statistic	Distribution
Mean	Known Variance	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$	$z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	Normal
Mean	Unknown Variance	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$	$t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ with $n - 1$ d.f.	Student-t
Mean	Comparison – Paired $d_i = x_{1i} - x_{2i}$	$H_0: \mu_1 - \mu_2 = \mu_d$ $H_1: \mu_1 - \mu_2 > \mu_d$	$t = \dfrac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$ with $n - 1$ d.f.	Student-t
Mean	Comparison – Independent Known variances	$H_0: \mu_1 - \mu_2 = \mu_d$ $H_1: \mu_1 - \mu_2 > \mu_d$	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - \mu_d}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	Normal
Mean	Comparison – Independent Unknown and unequal Variances	$H_0: \mu_1 - \mu_2 = \mu_d$ $H_1: \mu_1 - \mu_2 > \mu_d$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - \mu_d}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ with $\min(n_1 - 1, n_2 - 1)$ d.f. *	Student-t
Mean	Comparison – Independent Unknown but assumed equal variances	$H_0: \mu_1 - \mu_2 = \mu_d$ $H_1: \mu_1 - \mu_2 > \mu_d$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - \mu_d}{s\sqrt{(n_1)^{-1} + (n_2)^{-1}}}$ where $s = \sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$ with $n_1 + n_2 - 2$ df	Student-t
Proportion	Single	$H_0: p = p_0$ $H_1: p > p_0$	$z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ where $\hat{p} = \dfrac{r}{n}$	Normal
Proportion	Comparison	$H_0: p_1 = p_2$ $H_1: p_1 > p_2$	$z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $\hat{p} = \dfrac{r_1 + r_2}{n_1 + n_2}$, $\hat{p}_1 = \dfrac{r_1}{n_1}$, and $\hat{p}_2 = \dfrac{r_2}{n_2}$	Normal
Variance	Single	$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 > \sigma_0^2$	$\chi^2 = \dfrac{(n - 1)s^2}{\sigma_0^2}$ with $n - 1$ df	Chi-Square
Variance	Comparison	$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 > \sigma_2^2$	$F = \dfrac{s_1^2}{s_2^2}$ with num = $n_1 - 1$ df and denom = $n_2 - 1$ df	F

* Use Welch-Satterthwaite's formula for more accuracy:

$$df \approx \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2}$$