Pearson Curves – Statistics – Prof. Richard B. Goldstein

Type	Equation f(x) =	$\frac{1}{f}\frac{df}{dx} = \frac{x-a}{b_0 + b_1 x + b_2 x^2}$	Comments
I	$k\left(1+\frac{x}{a_1}\right)^{m_1}\left(1-\frac{x}{a_1}\right)^{m_2}$	$\frac{x(m_1+m_2)+(a_1 m_2 - a_1 m_1)}{(x+a_1)(x-a_2)}$	**beta-distribution** $-a_1 < x < a_2$, $m_1, m_2 > -1$
II	$k\left(1-\frac{x^2}{a^2}\right)^{m}$	$\frac{2mx}{x^2-a^2}$	$-a < x < a$, $m > -1$
III	$k\left(1+\frac{x}{a}\right)^{\mu a} e^{-\mu x}$	$\frac{-\mu x}{x+a}$	**gamma, chi-square** $-a < x < \infty$, μ, $a > 0$
IV	$k\left(1+\frac{x^2}{a^2}\right)^{-m} e^{-\mu \tan^{-1}(x/a)}$	$\frac{-2mx+a\mu}{x^2+a^2}$	$-\infty < x < \infty$, $a, \mu > 0$
V	$kx^{-q}e^{-a/x}$	$\frac{ax-q}{x^2}$	$0 < x < \infty$, $a > 0$, $q > 1$
VI	$kx^{-q_1}(x-a)^{q_2}$	$\frac{q_2}{x-a} - \frac{q_1}{x}$	**beta 2nd kind, Fisher-F** $a < x < \infty$, $q_1 < 1$, $q_2 > -1$, $q_1 > q_2 - 1$
VII	$k\left(1+\frac{x^2}{a^2}\right)^{-m}$	$\frac{-2mx}{x^2+a^2}$	$-\infty < x < \infty$, $m > ½$
VIII	$k\left(1+\frac{x}{a}\right)^{-m}$	$\frac{-m}{x+a}$	$-a < x \le 0$, $m > 1$
IX	$k\left(1+\frac{x}{a}\right)^{m}$	$\frac{m}{x+a}$	$-a < x \le 0$, $m > -1$
X	$ke^{-(x-m)/\sigma}$	$\frac{-1}{\sigma}$	**exponential** $m \le x < \infty$, $\sigma > 0$
XI	kx^{-m}	$\frac{-m}{x}$	**Pareto** $b \le x < \infty$, $m > 1$
XII	$k\left(\frac{1+\frac{x}{a_1}}{1-\frac{x}{a_2}}\right)^{m}$	$\frac{m(a_1+a_2)}{(a_2-x)(x+a_1)}$	$-a_1 < x < a_2$, $\|m\| < 1$
Normal	$ke^{-(x-\mu)^2/\sigma^2}$	$\frac{2(\mu-x)}{\sigma^2}$	$-\infty < x < \infty$

The most important curves for application are types I, III, VI, and VII.

Any Pearson curve is uniquely determined by the first four moments $\mu_r' = E(X^r) = \int_{-\infty}^{\infty} x^r f(x)\,dx$

The curves were introduced by Karl Pearson: *On the dissection of asymmetrical frequency curves* in 1894.

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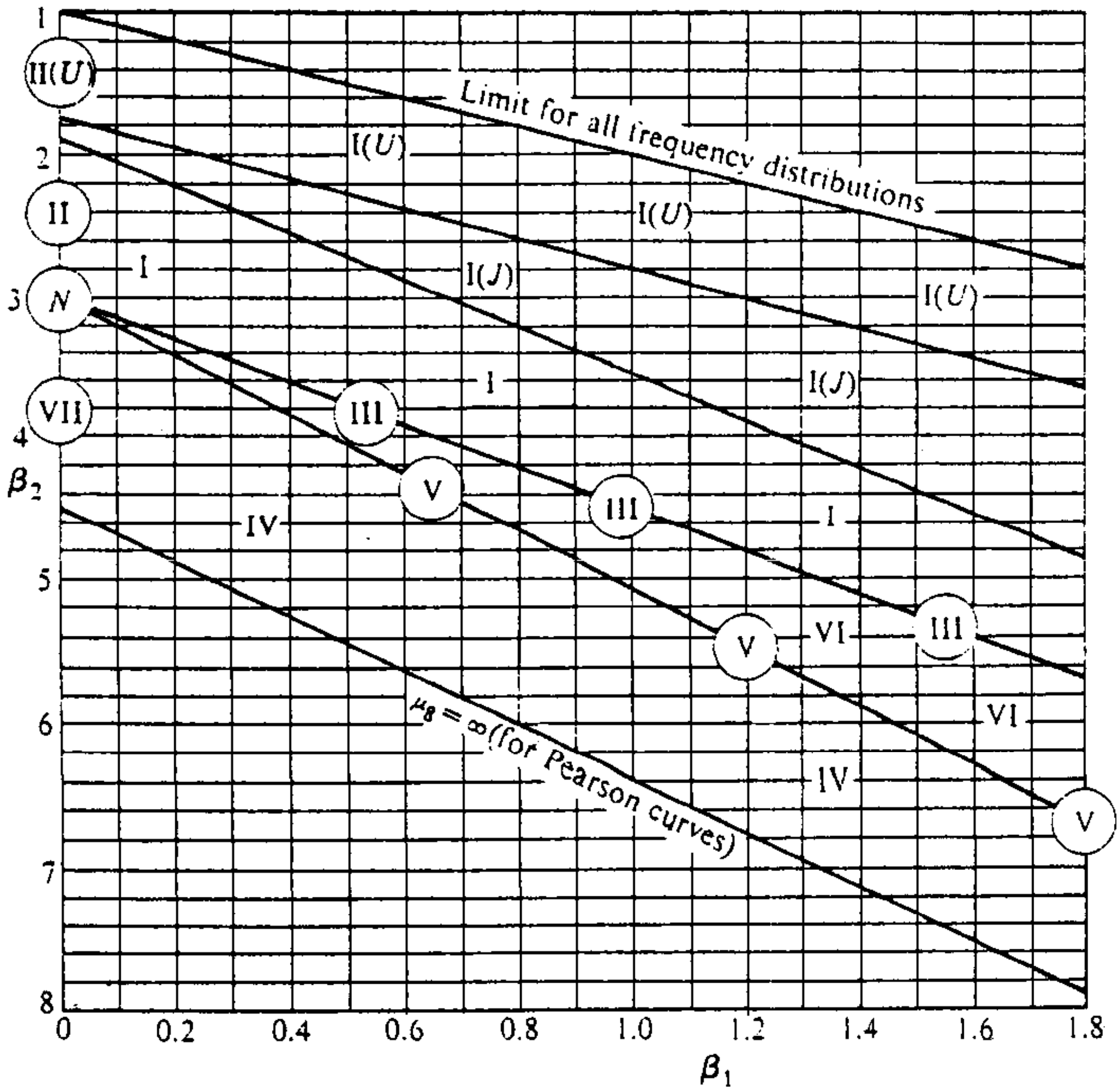

Figure 12.2 A Chart Relating the Type of Pearson Frequency Curve to the Values of β_1, β_2

The upside-down presentation of this figure is in accordance with well-established convention. Note that only Types I, VI, and IV correspond to areas in the (β_1, β_2) diagram. The remaining types correspond to lines and are sometimes called *transition types*. Other forms of diagrams have been proposed by Boetti (1964) and Craig (1936). The latter uses $(2\beta_2 - 3\beta_1 - 6)/(\beta_2 + 3)$ in place of β_2 for one axis.

Examples of fitting Pearson curves to numerical data are given in Elderton and Johnson (1969). Computer programs for producing values of random variables having Pearson type distributions have been described by Cooper et al. (1965).