Continuous Distributions – Prof. Richard B. Goldstein

$P(a < X < b) = \int_a^b f(x)\,dx$ where f(x) ≥ 0 for all x and $\int_{-\infty}^{\infty} f(x)\,dx = 1$ [probability density function]

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$ [cumulative distribution]

$$\mu = E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\sigma^2 = E\left[(X-\mu)^2\right] = E\left[X^2\right] - (E[X])^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$\mu_n = E\left[(X-\mu)^n\right] = \int_{-\infty}^{\infty} (x-\mu)^n f(x)\,dx \text{ [n}^{\text{th}}\text{ central moment]}$$

$$\mu_n{}' = E\left[X^n\right] = \int_{-\infty}^{\infty} x^n f(x)\,dx \text{ [n}^{\text{th}}\text{ moment about the origin]}$$

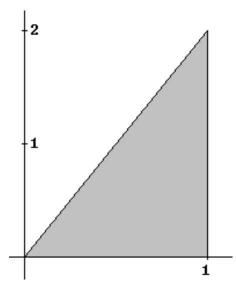
$$\mu_3 = E\left[(X-\mu)^3\right] = E\left[X^3 - 3\mu X^2 + 3\mu^2 X - \mu^3\right] = E\left[X^3\right] - 3\mu E\left[X^2\right] + 2\mu^3$$

$$\gamma_1 = \frac{\mu_3}{\sigma^3} \text{ [skewness]}$$

$$\mu_4 = E\left[(X-\mu)^4\right] = E\left[X^4 - 4\mu X^3 + 6\mu^2 X^2 - 4\mu^3 X + \mu^4\right] = E\left[X^4\right] - 4\mu E\left[X^3\right] + 6\mu^2 E\left[X^2\right] - 3\mu^4$$

$$\gamma_2 = \frac{\mu_4}{\sigma^4} - 3 \text{ [excess} - \text{kurtosis compared to normal]}$$

EXAMPLE: f(x) = 2x on [0, 1]



$$\mu = \int_0^1 x(2x)\,dx = \int_0^1 2x^2\,dx = \left.\frac{2x^3}{3}\right|_0^1 = \frac{2}{3}$$

$$\sigma^2 = \int_0^1 x^2(2x)\,dx - \left(\frac{2}{3}\right)^2 = \left.\frac{2x^4}{4}\right|_0^1 - \frac{4}{9} = \frac{1}{2} - \frac{4}{9} = \frac{1}{18} \approx 0.0556$$

$$\mu_3 = \int_0^1 x^3(2x)\,dx - 3\left(\frac{2}{3}\right)\int_0^1 x^2(2x)\,dx + 2\left(\frac{2}{3}\right)^3 = \left.\frac{2x^5}{5}\right|_0^1 - 2\left(\frac{2x^4}{4}\right)_0^1 + \frac{16}{27} = \frac{2}{5} - 1 + \frac{16}{27} = -\frac{1}{135}$$

$$\gamma_1 = \frac{-1/135}{(1/18)^{3/2}} \approx -0.566$$

Similarly $\mu_4 = \dfrac{1}{135}$ and $\gamma_2 = \dfrac{1/135}{(1/18)^2} - 3 = \dfrac{12}{5} - 3 = -\dfrac{3}{5}$