Bivariate / Joint Probability Distributions – Statistics – Prof. Richard B. Goldstein

Discrete $f(x,y) = P(X = x, Y = y) \geq 0 \ \forall x, y$

$$\sum_x \sum_y f(x,y) = 1 \quad P[(X,Y) \in A] = \sum_A \sum f(x,y)$$

Continuous $f(x,y) \geq 0 \ \forall x, y$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1 \quad P[(X,Y) \in A] = \iint_A f(x,y)\,dx\,dy$$

Marginal Distributions

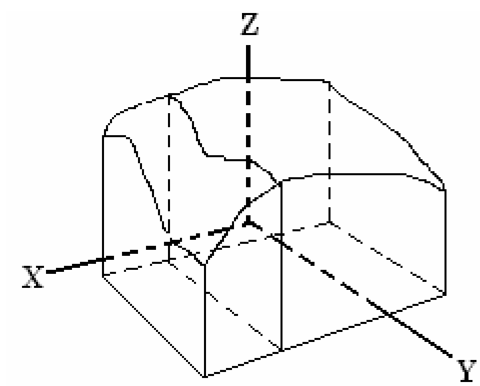
Discrete: $g(x) = \sum_y f(x,y)$ $\qquad h(y) = \sum_x f(x,y)$

Continuous: $g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$ $\qquad h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

Conditional Distributions

Y given X = x $\qquad f(y \mid x) = \dfrac{f(x,y)}{g(x)}, \ g(x) > 0$

X given Y = y $\qquad f(x \mid y) = \dfrac{f(x,y)}{h(y)}, \ h(y) > 0$



Y given X = x $\quad$ f(y|x)

Statistically Independent

$$\text{independent} \Leftrightarrow f(x,y) = g(x)h(y) \ \forall x, y$$

Expected Values and Variances (similar formulas for discrete case)

$$E[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\,dx\,dy \qquad E[Y] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f(x,y)\,dx\,dy$$

$$E[X^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x,y)\,dx\,dy \qquad \mathrm{Var}[X] = E[X^2] - (E[X])^2$$

$$E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f(x,y)\,dx\,dy \qquad \mathrm{Co\,var}[X,Y] = E[XY] - E[X]E[Y]$$

$$E[Y \mid X = x] = \int_{-\infty}^{\infty} y f(y \mid x)\,dy \qquad E[X \mid Y = y] = \int_{-\infty}^{\infty} x f(x \mid y)\,dy$$