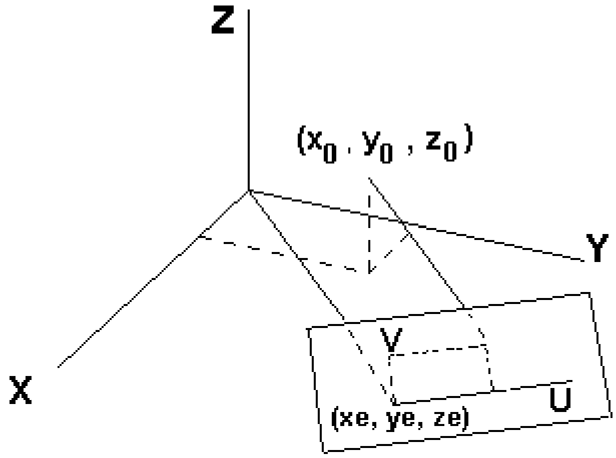
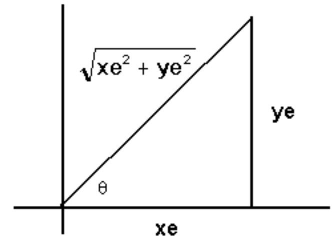
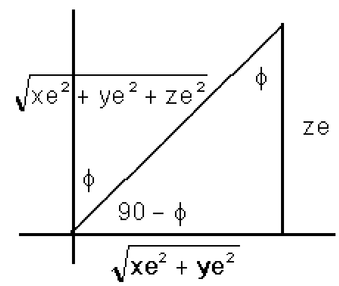
2-D Rendering of 3-D Objects
Projection and Perspective
Prof. Richard B. Goldstein







To go from a 3-D coordinate system of (x, y, z) to a 2-D coordinate system of (u, v) two rotations are made. The first is a rotation of angle $90^o + \theta = \pi/2 + \theta$ around the z-axis and the second is a rotation of $-\varphi$ around the x-axis. The new coordinates are found by multiplying two matrices:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\varphi \\ 0 & -\sin\phi & \cos\varphi \end{bmatrix} \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\cos\theta & -\sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\cos\phi\cos\theta & -\cos\phi\sin\theta & \sin\phi \\ \sin\phi\cos\theta & \sin\phi\sin\theta & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Since

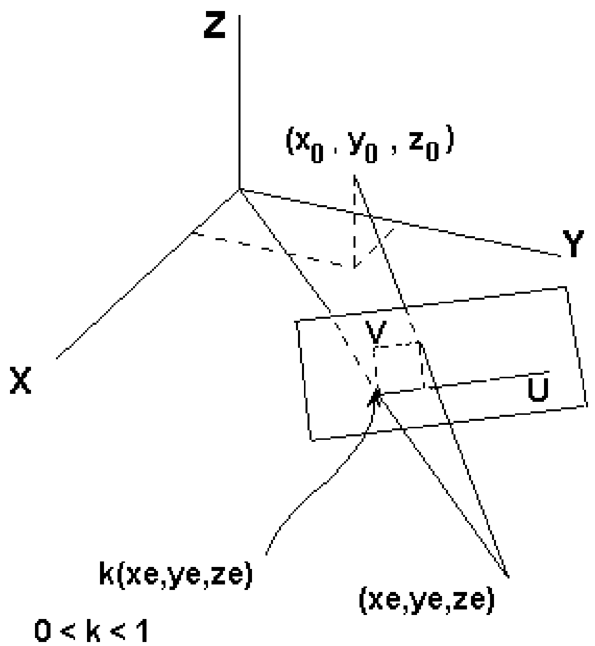
$$\cos\theta = \frac{xe}{\sqrt{xe^2 + ye^2}}, \sin\theta = \frac{ye}{\sqrt{xe^2 + ye^2}}, \cos\varphi = \frac{ze}{\sqrt{xe^2 + ye^2 + ze^2}}, \quad \text{and } \sin\varphi = \frac{\sqrt{xe^2 + ye^2}}{\sqrt{xe^2 + ye^2 + ze^2}}$$

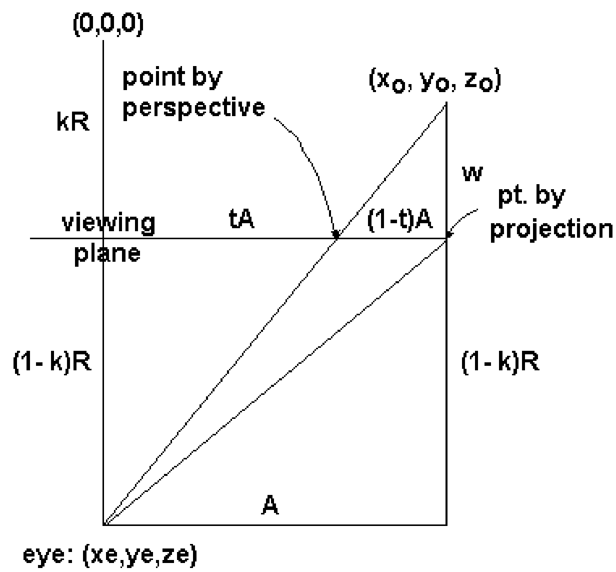
$$\boxed{u = -x_0 \sin\theta + y_0 \cos\theta = \frac{-x_0 * ye + y_0 * xe}{\sqrt{xe^2 + ye^2}}}$$

$$\boxed{v = -x_0 \cos\theta \cos\varphi - y_0 \sin\theta \cos\varphi + z_0 \sin\varphi = \frac{-(x_0 * xe + y_0 * ye) * ze + z_0 (xe^2 + ye^2)}{\sqrt{xe^2 + ye^2}\sqrt{xe^2 + ye^2 + ze^2}}}$$

Using screen graphics with the origin at (cx, cy) and scale=s the new point is located at (cx + s*u, cy – s*v). Remember, the y-axis is upside down in screen graphics.

If the picture is in perspective, then use the following





Using the common ratios in the triangle shown on the right the (u, v) coordinates are reduced by a factor of t. The distance from the point (x_0, y_0, z_0) to the viewing plane is w (assume w > 0).

$$\frac{(1-t)A}{w} = \frac{A}{(1-k)R + w}$$

$$\text{where } R = \sqrt{xe^2 + ye^2 + ze^2} \text{ and } w = \frac{k(R^2) - x_0 * xe - y_0 * ye - z_0 * ze}{R}$$

$$t = 1 - \frac{w}{(1-k)R + w} = \frac{1-k}{1 - \left(\dfrac{x_0 * xe + y_0 * ye + z_0 * ze}{xe^2 + ye^2 + ze^2}\right)}$$

which gives the new u and v coordinates as:

$$u = t * \left(\frac{y_0 * xe - x_0 * ye}{\sqrt{xe^2 + ye^2}}\right)$$

$$v = t * \left(\frac{\left(xe^2 + ye^2\right)z_0 - (x_0 * xe + y_0 * ye) * ze}{R\sqrt{xe^2 + ye^2}}\right)$$

Note: k only affects the scaled size of objects, not their shape.