Parametric Representation of Surfaces – Prof. Richard B. Goldstein

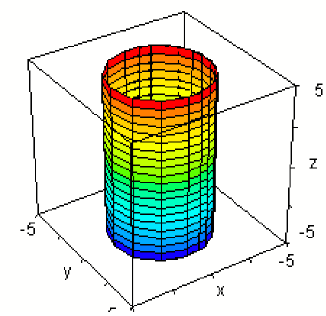
r(u, v) = x(u, v) **i** + y(u, v) **j** + z(u, v) **k**

Normal Vector $\mathbf{r}_u \times \mathbf{r}_v$

$$A(s) = \iint_D |r_u \times r_v| \, dA = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \, dA$$

Examples

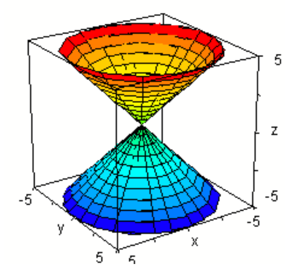
Cylinder



x = R cos u, y = R sin u, z = v

$x^2 + y^2 = R^2$

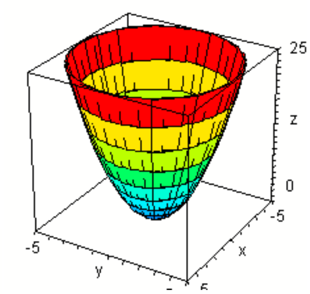
Cone



x = u cos v, y = u sin v, z = ku

$z^2 = k^2(x^2 + y^2)$

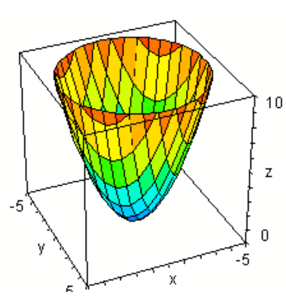
Paraboloid



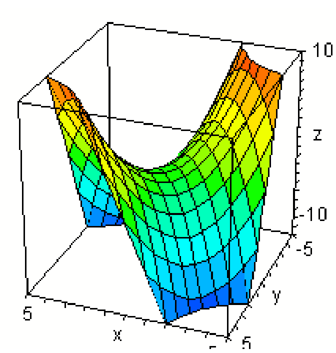
x = u cos v, y = u sin v, z = u^2

$z = x^2 + y^2$

(alt) x = u + v, y = u – v, z = $2u^2 + 2v^2$
notice – difference in grid lines
$z = x^2 + y^2$



Hyperbolic Paraboloid



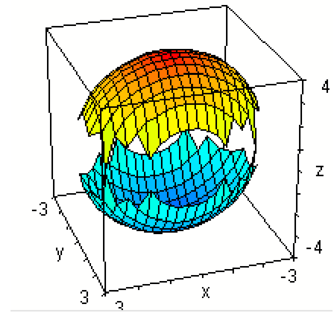
x = u + v, y = u – v, z = $u^2 - v^2$

z = xy

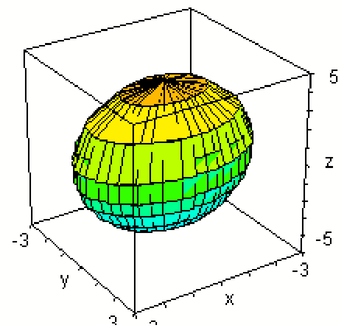
Ellipsoid

$$\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{16}=1$$

Using $z=\pm 4\sqrt{1-\frac{x^2}{9}-\frac{y^2}{4}}$ (cracked egg?)

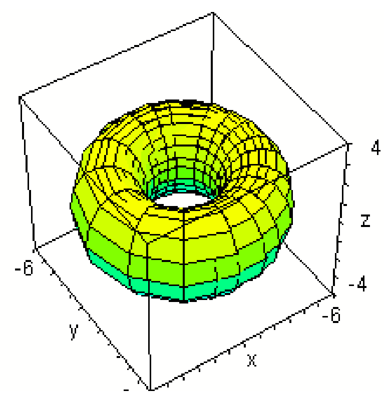


Better: x = 3 sin u cos v, y = 2 sin u sin v, z = 4 cos u



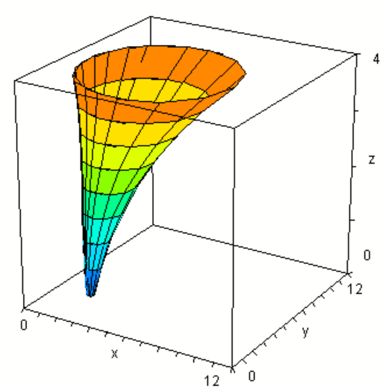
Torus (donut)

x = 4 cos u + 2 cos u cos v
y = 4 sin u + 2 sin u cos v
z = 2 sin v



Tornado

$x = 2 + 0.2v^2 + (e^{0.4v} - 0.8)\cos u$
$y = 3 + 0.4v + (e^{0.4v} - 0.8)\sin u$
$z = v$



Spaceship

x = cos u sin v
y = sin u sin v
z = cos v + ln(tan(v/2))